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THE MATHEMATICAL MODEL OF HEAT TRANSFER IN MATLAB FOR DEVICE OF CARDIO-PULMONARY BYPASS IN CARDIOLOGY

Запропоновано модель теплопередачі для пристрою штучного кровообігу у вигляді системи диференціальних рівнянь. Чисельна модель дозволяє оцінити температурні параметри гіпотермії і гіпертермії серця і досліджувати зміни градієнта температури на поверхні міокарда в момент реєстрації теплових зображень серця.

Ключові слова: термограма; міокард; розподіл температури; судинна патологія.

Предложена модель теплопередачи для устройства искусственного кровообращения в виде системы дифференциальных уравнений. Численная модель позволяет оценить температурные параметры гипотермии и гипертермии сердца и исследовать изменения градиента температуры на поверхности миокарда в момент регистрации тепловых изображений сердца.

Ключевые слова: термограмма; миокард; распределение температуры; сосудистая патология.

The heat transfer models for device of cardiopulmonary bypass in the form of a system of ordinary differential equations are presented. The numerical model allows us to estimate the temperature of the process parameters of hypothermia and hyperthermia heart and to investigate changes in the gradient of the temperature on the surface of the myocardium at the time of registration of thermal images of the heart.

The numerical model of heat transfer during extracorporeal circulation in the form of a system of ordinary differential equations was created. It allows to estimate the parameters of temperature propagation and temperature gradients at the surface of the myocardium at the time of registration of thermal images of the heart.

Comparison of the model with real patients intraoperative infra-red thermography shows that this method can provide additional important information regarding temperature and vascular uniformity in time of myocardial cooling and heating. This method increasing safety of myocardium protection and assistive in solutions of weaning from cardiopulmonary bypass.

Keywords: thermogram; myocardium; temperature distribution; vascular pathology.

Introduction. The heat transfer during extracorporeal cardiopulmonary bypass (CPB) is due to the heat exchange between the blood and the water in the heat exchanger device of cardiopulmonary bypass (DCB) and to the heat exchange between the blood and the body of the patient's in the circulatory system. In accordance with the protocol of CPB blood is first using a special catheter enters the oxygenator to the pump (centrifuge pump) which replaces lung function, and then from the oxygenator blood moves to the heat exchanger, which lowers the blood temperature and across a catheter (silicone tube) directed in the patient's system circulation [1].

For the CPB the blood flow is maintained at $2,2 - 2,4 \frac{L}{\min \cdot m^2}$ and temperature at 28 – 35 ºC during moderate hypothermia, and 16 – 26 ºC during deep hypothermia. Reduction of the patient's body temperature leads to reduced oxygen demand for bodies. To perform the heat balance the heat exchange in the body must be equal to its heat transfer. At rest, the magnitude of the human body heat is about $75 kcal/g$ at about 36,6 ºC. When heating blood the amount of heat that is absorbed or excreted of blood, calculated using the formula:

$$Q_{lic} = c_{liq} m_{liq} (T_{lic} - T_{out}), J,$$

where c_{liq} – the specific heat capacity of the blood, $c_{liq} \approx 4200 \frac{J}{kg \cdot s}$, $(T_{lic} - T_{out})$ – the temperature difference at heating blood, $°C$, m_{liq} – the mass of cooled blood, kg.

The heat balance equation involving DCB for cooling and heating of blood will look:

$$Q_{lic} + Q_{heart} = Q_{dbc},$$

where Q_{heart} – the amount of heat emitted or absorbed by the myocardium (heart) for cooling or heating of blood, J. Q_{dbc} – the amount of heat that is released or absorbed in the DCB, J.

Obviously, the combustion of the blood the temperature difference is $(T_{lic} - T_{out}) > 0$ and correspondingly the amount of heat is $Q_{lic} > 0$, and when the cooling blood the temperature difference is $(T_{lic} - T_{out}) < 0$ and $Q_{lic} < 0$ – blood releases energy in the DCB.

The terms in equation of the heat balance describe the contribution to the energy balance of the environment: the blood, the heart, the body, the air. The amount of heat which is transferred from the inner layer to the outer layer of the myocardium is described of the Fourier law [2]:

$$Q_{heart} = -K \frac{T_2 - T_1}{l} S_{heart} \cdot t,$$

$T_1 = T_{heart} = \dfrac{T_1' + T_2'}{2}$ –the temperature of the inner wall of the heart; $T_1' = T_0$ –the initial value of the blood temperature in the DCB; $T_2' = T_{liq}$ – the final cooling temperature of blood in the DCB; K – the thermal conductivity of the myocardium; S_{heart} – the area of the myocardium; l – the thickness of the myocardium.

The amount of heat that must be expended to cool the blood in the DCB from T_1' to T_2' can be found from the expression:

$$Q_{dcb} = \left(m_{lic} \cdot c_{lic} + m_{dcb} \cdot c_{dcb}\right) \cdot \left(T_1' - T_2'\right),$$

m_{lic}, m_{dcb} – the mass of blood and the mass of material in the DCB, respectively, kg, c_{lic} – the specific heat capacity of the blood, $\frac{J}{kg \cdot K}$, c_{bca} – the specific heat capacity of the material in the DCB, $\frac{J}{kg \cdot K}$.

Thus, it is possible to obtain a mathematical model in the form of a conventional differential equation of the 1st-order that describes the heat exchange between the blood and the heart:

$$\begin{cases} \dfrac{dQ_{heart}}{dt} = \dfrac{1}{R_q}\left(T_{heart} - T_{out}\right) \\[2mm] \dfrac{dT_{heart}}{dt} = -r(T_0 - T_{lic}) \end{cases},$$

where $T_0 = T_{heart}\mid_{t=0}, K$ at time $t = 0$. $R_q = \dfrac{1}{\alpha}$ –the thermodynamic resistance of the myocardium, $\frac{K}{W}$,

$r = \dfrac{1}{(m_{lie}c_{lie} + m_{dcb}c_{dcb}) \cdot R_q}$ –the coefficient cooling of the myocardium, $T_{out} = T_{heart}\mid_{t_{i-1}}, K$ – the temperature in the preceding step $(i-1)$ the heat exchangefor time $t = t_{i-1}$.

Given the temperature blood in the DCB and tissue myocardium, the temperature body of the patient and the air in the operating room, the model of heat exchange can be represented as the system of ordinary differential equations: the system (1) describes the heat exchange between he fluids blood-DCB, the system (2) describes the heat exchange between the fluids blood-heart, system (3) describes the heat exchange between the fluids air-body and the fluids body-heart.

Thus, it is the model of heat exchange is described systems of the differential equations:

$$\begin{cases} \dfrac{dQ_{heart}}{dt} = \dfrac{1}{R_q}\left(T_{heart} - T_{out1}\right), \\[3mm] \dfrac{dT_{heart}}{dt} = \dfrac{1}{(m_{lic}c_{lic} + m_{bca}c_{bca})}\left(\dfrac{dQ_{lic}}{dt} - \dfrac{dQ_{heart}}{dt}\right) - \dfrac{dT_{lic}}{dt} \end{cases}, \quad (1)$$

$$\begin{cases} \dfrac{dQ_{lic}}{dt} = \dfrac{1}{R_q}\left(T_{lic} - T_{out2}\right), \\[3mm] \dfrac{dT_{lic}}{dt} = \dfrac{1}{(m_{lic}c_{lic} + m_h c_h)}\left(\dfrac{dQ_{heart}}{dt} - \dfrac{dQ_{lic}}{dt}\right) \end{cases}, \quad (2)$$

$$\begin{cases} \dfrac{dQ_{body}}{dt} = \dfrac{1}{R_q}\left(T_{body} - T_{out3}\right), \\[3mm] \dfrac{dT_{body}}{dt} = \dfrac{1}{(m_{lic}c_{lic} + m_b c_b)}\left(\dfrac{dQ_{heart}}{dt} - \dfrac{dQ_{body}}{dt}\right), \\[3mm] \dfrac{dQ_{air}}{dt} = \dfrac{1}{R_q}\left(T_{air} - T_{out4}\right), \\[3mm] \dfrac{dT_{air}}{dt} = \dfrac{1}{(m_{air}c_{air} + m_b c_b)}\left(\dfrac{dQ_{body}}{dt} - \dfrac{dQ_{air}}{dt}\right) \end{cases}, \quad (3)$$

The temperatures $T_{out1}, T_{out2}, T_{out3}, T_{out4}$ correspond to the values temperature at the previous step $(i-1)$ for the heat exchange between the blood and the objects to interact (DCB material, myocardium, body, air).

The model of the heat transfer for cardiopulmonary bypass

According to the above model of the heat transfer for extracorporeal cardiopulmonary bypass, the program model in Matlab has the form shown in Fig. 1.

The program model in Matlab for each system of the differential equations

$$\frac{dQ}{dt} = \frac{1}{R_q}\left(T_1 - T_2\right),$$

and

$$\frac{dT}{dt} = \frac{1}{(m_1 c_1 + m_2 c_2)}\left(\frac{dQ_1}{dt} - \frac{dQ_2}{dt}\right),$$

at using Simulink library elements has the form shown in Fig. 2.

The function of approximate lowering blood temperature by heat exchange in the DCB has the form shown in Fig. 3.
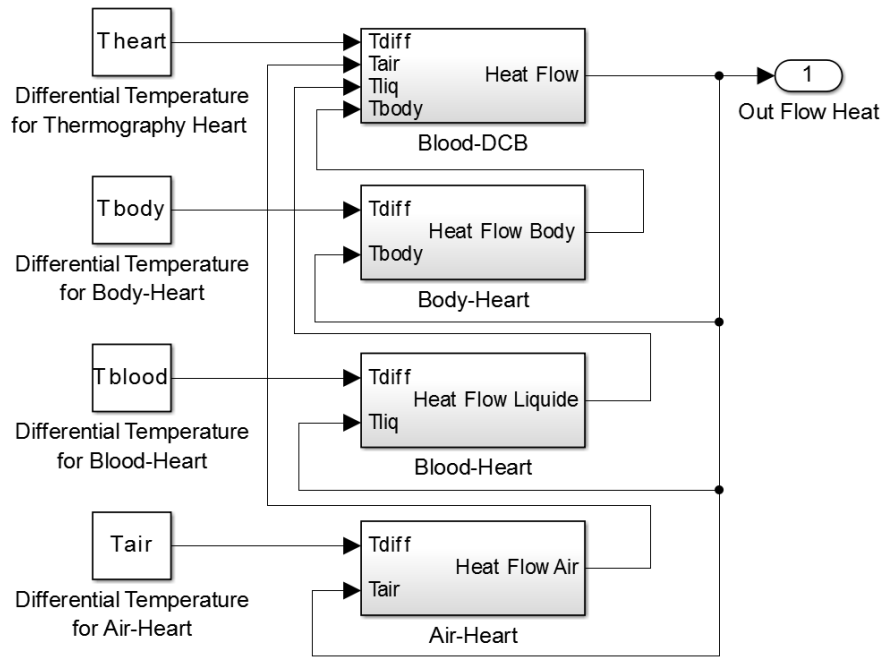
Fig. 1 – Block diagram of the mathematical model of the heat transfer for the process of cooling blood in Matlab
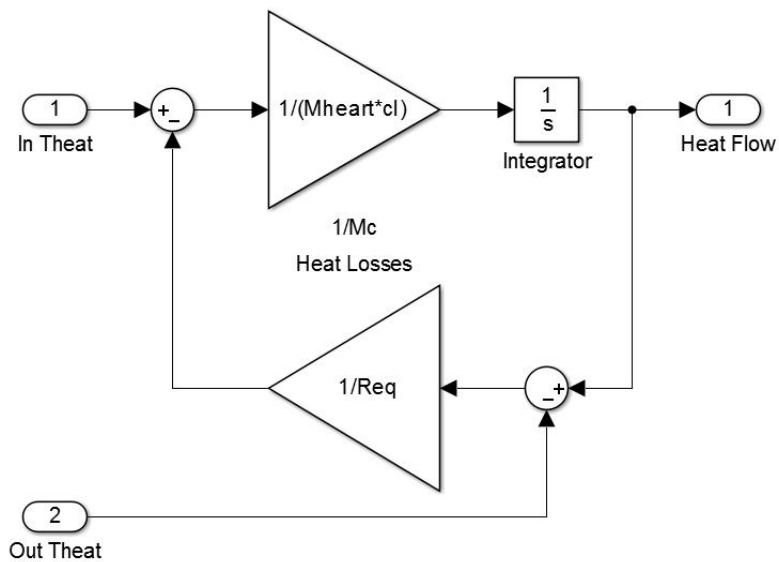

Fig. 2 – Block diagram of the mathematical model of the heat transfer for each system of the differential equations in Matlab
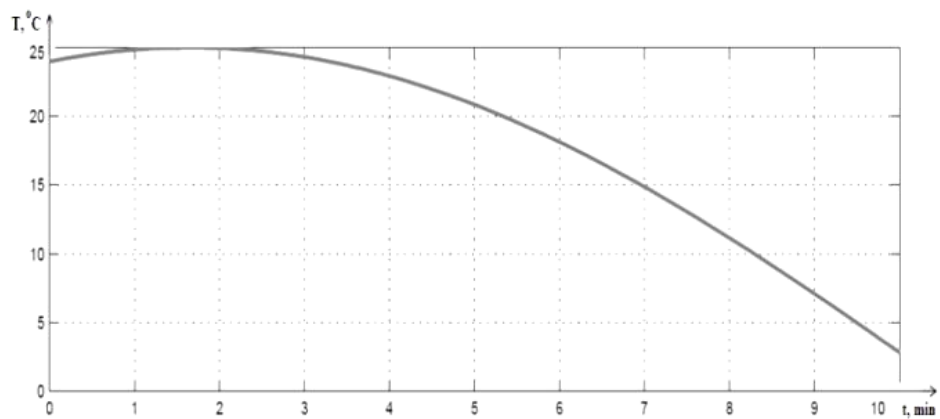

Fig. 3 – The process of step lowering blood temperature by heat exchange

The programming model of heat transfer that calculated using DCB function allows assessing the depth of cooling and rewarming of the heart at any given time during extracorporeal cardiopulmonary bypass [3].

The verification of heat transfer model. For experimental verification of heat transfer model with a thermal imager Flir i7 at intervals of 1 minute were recorded thermographic images of the myocardium of the heart in the process of cooling.The initial and final stage of the cooling process of the heart in the form of infrared images of the myocardium and the surgical field displayed in Fig. 4, a, b:

Fig. 4 – Heart with temperature of cooling myocardium from 33,6 °C to 10,3 °C : a – the heart temperature before cooling; b – the heart temperature during cooling; c – thermogram of the heart during cooling

At the beginning of hypothermia process the temperature of area for the heated myocardium was 33,6 ºC and in the final stage was lowered to 10.3 ºC. The indications DCB at the end of the process hypothermia are determined that the temperature of open-heart surgery was maintained at 17 ºC.The temperature distribution on the surface of the myocardium during cooling the cardiac at the readings DCB equal 17 ºC is shown in Fig. 5.
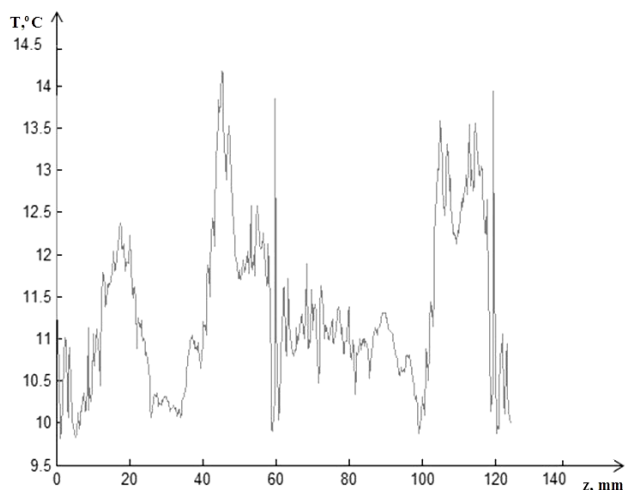

Fig. 5 – The temperature distribution on the surface of the myocardium upon cooling to 14 – 10 ºC

The temperature profile is constructed for chilled the heart and indicates the minimum temperature of myocardium 9,8–10,3 ºC and the greatest difference between the maximum and minimum temperatures of around 3–4 ºC. Therefore, temperature gradient on the surface of the myocardium before and after cooling of the heart is stable indicator which probably can be used as a diagnostic criterion in determining ischemic areas on the surface of the myocardium.

Conclusions

Thus, the heat balance model that developed for extracorporeal cardiopulmonary bypass allowed the evaluate the dynamics of the cooling process and warming the heart and determine the temperature gradient at the surface of the myocardium during the registration process thermo grams hypo- and hyperthermia and obtain the distribution of temperature profiles, which enables diagnosis of ischemic lesions in the myocardium.

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